

Intrinsic and extrinsic origins of the polar Kerr effect in a chiral p -wave superconductor

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Abstract

Recently, the measurement of the polar Kerr effect (PKE) in the quasi two-dimensional superconductor Sr_2RuO_4 , which is motivated to observe the chirality of $p_x + ip_y$ -wave pairing, has been reported. We clarify that the PKE has intrinsic and extrinsic (disorder-induced) origins. The extrinsic contribution would be dominant in the PKE experiment.

1 Introduction

The polar Kerr effect (PKE) in which the direction of polarization of reflected linearly polarized light is rotated is renowned as a phenomenon of the light in ferromagnetic compounds[1]. Recent measurement on the PKE in the quasi two-dimensional superconductor Sr_2RuO_4 [2] is motivated to observe the chirality of $p_x + ip_y$ -wave state, which is the most plausible pairing symmetry in this superconductor[3].

We suppose that $z > 0$ is empty and $z < 0$ filled by the superconductor and incident polarized light propagates along z direction. The Kerr rotation angle is given by [1, 4]

$$\theta_K = -\text{Im} \left[\frac{\omega(q_z^+ - q_z^-)}{\omega^2 - q_z^+ q_z^-} \right]. \quad (1)$$

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where $q_z^\pm = \sqrt{\omega^2 + i\omega\sigma_{xx}(\omega) \pm \omega\sigma_{xy}(\omega)}$, which is the dispersion relation of light inside the superconductor, and σ_{xx} and σ_{xy} are the diagonal and Hall parts of the conductivity tensor, respectively. It is well established that $\sigma_{xx}(\omega)$ in the high frequency regime $\omega \gg 2|\Delta|$ ($|\Delta|$; amplitude of the gap function) is given by the Drude form $\sigma_{xx}(\omega) = \omega_p^2\tau_0(1 - i\omega\tau_0)^{-1}$, where ω_p is the plasma frequency and τ_0 quasiparticle life time[5, 6].

We see from the numerator of Eq. (1) that the Hall conductivity is crucial to the PKE. In the following sections we discuss intrinsic and extrinsic parts of the Hall conductivity and obtain the Kerr angle (the terminologies “intrinsic” and “extrinsic” has been originally used in the anomalous Hall effect and spin Hall effect [7]). To obtain the Hall conductivity, we use the Kubo formula

$$\sigma_{xy}(\omega, \mathbf{q}_\perp) = \frac{1}{2i\omega} \epsilon_{ij} \langle J_i(\omega, \mathbf{q}_\perp) J_j(-\omega, -\mathbf{q}_\perp) \rangle, \quad (2)$$

where $\mathbf{q}_\perp = (q_x, q_y)$ and $\mathbf{J}(\omega, \mathbf{q}) = -e \sum_{\mathbf{p}} \frac{\mathbf{p} + \mathbf{q}_\perp}{2m_e} \Psi_{\mathbf{p}}^\dagger \Psi_{\mathbf{p} + \mathbf{q}_\perp}$ is the electric current.

2 Intrinsic Hall conductivity and PKE: Clean limit

We consider the clean limit in this section. We start from a single band model which would describe the γ -band in Sr_2RuO_4 . The Hamiltonian for the Nambu Fermion field $\Psi_{\mathbf{p}} = (c_{\mathbf{p}\uparrow}, c_{-\mathbf{p}\downarrow}^*)^T$ is

$$\hat{H} = \begin{pmatrix} \epsilon(\mathbf{p} + e\mathbf{A}) & \Delta_{\mathbf{p}} \\ \Delta_{\mathbf{p}}^* & -\epsilon(\mathbf{p} - e\mathbf{A}) \end{pmatrix}, \quad (3)$$

where $\epsilon(\mathbf{p}) = \frac{p_x^2 + p_y^2}{2m_e} - \epsilon_F$ and $\Delta_{\mathbf{p}} = \Delta(\hat{p}_x + i\hat{p}_y)$. We should note that, since the Cooper pair has the electric charge $-2e$, there is the phase field θ of the gap function ($U(1)$ Goldstone mode) $\Delta_{\mathbf{p}} = \tilde{\Delta}_{\mathbf{p}} e^{-2ie\theta}$, where $\tilde{\Delta}_{\mathbf{p}}$ is the gauge invariant component of the gap function since the phase field restores the gauge invariance. We can obtain the manifestly gauge-invariant current-current correlation by integrating out $\Psi_{\mathbf{p}}$ and also θ [8, 11, 12] (see also Appendix). Then, using Eq. (2), we have the intrinsic Hall conductivity

$$\begin{aligned} \sigma_{xy}^{(I)}(\omega, \mathbf{q}_\perp) &= a(\omega, \mathbf{q}_\perp) \frac{-v_F^2 \mathbf{q}_\perp^2}{\omega^2 - v_F^2 \mathbf{q}_\perp^2} \\ a(\omega, \mathbf{q}_\perp) &= \frac{e^2}{4\pi d} N_w f^{(I)}(\omega, \mathbf{q}_\perp), \end{aligned} \quad (4)$$

where $d \simeq 6.6\text{\AA}$ is the inter-layer distance and $f^{(I)}(\omega, \mathbf{q}_\perp)$ is a temperature-dependent regular function that satisfies $f^{(I)}(0, 0) = 1$ [9, 10, 11, 12], and θ -integral yields a singular factor $\frac{-v_F^2 \mathbf{q}_\perp^2}{\omega^2 - v_F^2 \mathbf{q}_\perp^2}$. The coefficient

$$N_w = \int \frac{d^2 p}{4\pi} \hat{\mathbf{g}}_{\mathbf{p}} \cdot (\partial \hat{\mathbf{g}}_{\mathbf{p}} \times \partial \hat{\mathbf{g}}_{\mathbf{p}})$$

is the winding number first suggested by Volovik[13] that equals to the chirality of the Cooper pair represented by the internal angular momentum l_z (for the chiral p -wave state, $l_z = 1$). See, also [8, 14, 15].

In the measurement [2], the incident beam was set up as $\mathbf{q} = \hat{\mathbf{z}}q_z$, then, $\sigma_{xy}^{(I)}(\omega, \mathbf{q}_\perp) = 0$. In the recent papers [11, 12], the lens effect that focuses the light to a spot on the sample surface causes $\mathbf{q}_\perp \neq 0$ has been taken into account and found a finite $\sigma_{xy}(\omega, \mathbf{q}_\perp)$, but the estimated Kerr angle is about 9 orders of magnitude smaller than the observed Kerr angle.

It has been pointed out that an in-plane incident light could lead a rather large Kerr angle[16].

3 Extrinsic Hall conductivity and PKE

We shall take into account impurity scattering and obtain the extrinsic Hall conductivity. In this section, we neglect the intrinsic contribution. By integrating out the Fermion field[17] and the phase field θ , we obtain the gauge-invariant current-current correlation function. By using the Kubo formula Eq. (2), the extrinsic part can be summarized in the general form (see Appendix),

$$\sigma_{xy}^{(E)}(\omega, \mathbf{q}_\perp) = b(\omega, \mathbf{q}_\perp) \frac{\omega^2}{\omega^2 - v_F^2 \mathbf{q}_\perp^2}, \quad (5)$$

where a singular factor $\frac{\omega^2}{\omega^2 - v_F^2 \mathbf{q}_\perp^2}$ comes from the integration of the phase field. In the experimental situation, $\frac{\omega^2}{\omega^2 - v_F^2 \mathbf{q}_\perp^2} \simeq 1$ and the extrinsic part would be dominant.

The lowest-order contribution to the coefficient $b(\omega, \mathbf{q}_\perp)$ is from the current-current correlation function with the vertex correction by a single delta-function type disorder within the second Born approximation, which is in the third order of the impurity strength.[17]. Although the diagram looks similar to the skew scattering diagram in the anomalous Hall effect in a ferromagnet[18], the critical difference is that the scatterer we consider does not include the spin-orbit interaction.

The extrinsic mechanism was first pointed out by Ref. [17]. Recently, the frequency dependence of the result was corrected by Ref. [19]. Additionally, a contribution from the ladder sum of the scatterings in the first Born approximation is investigated in detail[19].

Although quantitative agreement with the experiment has not yet been obtained, we believe that the extrinsic mechanism would be the essence of the PKE in Sr_2RuO_4 . More consideration would be needed for the estimation of $b(\omega, \mathbf{q})$.

4 Summary

We have pointed out that the PKE in a chiral p -wave superconductor has intrinsic and extrinsic (disorder-induced) origins. The general forms of these contributions has been clarified (See, Eqs. (4) and (5). See, also Eq. (12) in Appendix). The intrinsic mechanism comes from the chirality of the chiral p -wave pairing state, and the extrinsic one comes from the combination of the pairing state and the delta-function type disorder. The latter would be the essence of the PKE observed in Sr_2RuO_4 [2].

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Appendix

The aim of this appendix is to derive the manifestly gauge-invariant current-current correlation function and obtain the Hall conductivity by using Eq. (2). For simplicity, we consider the zero temperature. The extension to the finite temperature would be straightforward. First, we integrate out Fermion from Eq. (3). The effective Lagrangian for gauge fields and the phase field (in the Gaussian approximation) is [8, 11, 12, 13]

$$\begin{aligned} \mathcal{L}_{eff}[A_\mu, \theta] = & \frac{1}{2\lambda_L(q)^2 v_F^2} (A_0(q) + iq_0\theta(q))(A_0(-q) - iq_0\theta(-q)) \\ & - \frac{1}{2\lambda_L(q)^2} (A_i(q) + iq_i\theta(q))(A_i(-q) - iq_i\theta(-q)) \end{aligned}$$

$$\begin{aligned}
& -\frac{a(q)}{2}i\epsilon_{ij}((A_0(q) + iq_0\theta(q))q_iA_j(-q) + A_i(q)q_j(A_0(-q) - iq_0\theta(q)) \\
& + \frac{b(q)}{2}i\epsilon_{ij}A_i(q)q_0A_j(-q),
\end{aligned} \tag{6}$$

where $q_\mu = (q_0, \mathbf{q}_\perp) = (\omega, \mathbf{q}_\perp)$. The first and second terms correspond to the coulomb screening term and Meissner term in the low-frequency and long-wavelength limit. The chiral p -wave pairing yields the third and the last terms. They are T-odd anomalous terms and in the low-frequency and long-wavelength limit, we refer to them as "Chern-Simons-like term". The last term vanishes in the clean limit[8, 11, 12, 13, 14, 15], while becomes nonzero in a disordered system[17]. Eq. (6) is in the gaussian form and we can easily integrate out the phase field θ . After the integration, we have,

$$\int \mathcal{D}\theta \exp(i\mathcal{L}_{eff}[A_\mu, \theta]) = \exp(i\mathcal{L}'_{eff}[A_\mu]), \tag{7}$$

where

$$\mathcal{L}'_{eff}[A_\mu] = \frac{1}{2}A_\mu(q)\Pi_{\mu\nu}(q)A_\nu(-q), \tag{8}$$

and

$$\Pi_{00}(q) = \frac{-v_F^2\mathbf{q}_\perp^2}{\omega^2 - v_F^2\mathbf{q}_\perp^2} \frac{1}{\lambda_L(q)^2 v_F^2}, \tag{9}$$

$$\begin{aligned}
\Pi_{0j}(q) &= \Pi_{j0}(-q) \\
&= -ia(q)\epsilon_{ij}q_i + \frac{\omega}{\omega^2 - v_F^2\mathbf{q}_\perp^2} \left(\frac{q_j}{\lambda_L(q)^2} - i(a(q) - b(q))\epsilon_{kj}\omega q_k \right),
\end{aligned} \tag{10}$$

$$\begin{aligned}
\Pi_{ij}(q) &= \frac{-1}{\lambda_L(q)^2}\delta_{ij} + ib(q)\epsilon_{ij}\omega \\
&\quad - \frac{\lambda_L(q)^2 v_F^2}{\omega^2 - v_F^2\mathbf{q}_\perp^2} \left(\frac{q_i}{\lambda_L(q)^2} - i(a(q) - b(q))\epsilon_{ki}\omega q_k \right) \left(\frac{q_j}{\lambda_L(q)^2} + i(a(q) - b(q))\epsilon_{lj}\omega q_l \right).
\end{aligned} \tag{11}$$

are the density-density, density-current, and current-current correlation functions, respectively. We see that they satisfy the condition for gauge invariance (charge conservation); $q_\mu \Pi_{\mu\nu} = \Pi_{\mu\nu} q_\nu = 0$. From the Kubo formula Eq. (2), we obtain the Hall conductivity

$$\begin{aligned}
\sigma_{xy}(\omega, \mathbf{q}_\perp) &= \frac{\epsilon_{ij}}{2i\omega} \Pi_{ij}(\omega, \mathbf{q}_\perp) \\
&= \frac{-v_F^2\mathbf{q}_\perp^2}{\omega^2 - v_F^2\mathbf{q}_\perp^2} a(\omega, \mathbf{q}_\perp) + \frac{\omega^2}{\omega^2 - v_F^2\mathbf{q}_\perp^2} b(\omega, \mathbf{q}_\perp),
\end{aligned} \tag{12}$$

where the first term corresponds to the intrinsic part Eq. (4) and the second term corresponds to the extrinsic part Eq. (5). When $b(\omega, \mathbf{q}_\perp) = 0$, the result agrees with the literature discussing the clean limit[8, 11, 12].

In the PKE measurement[2], $\mathbf{q} = \hat{\mathbf{z}}q_z$, and then, $\sigma_{xy}(\omega) = b(\omega)$. This fact indicates that the induction of the coefficient $b(\omega)$, namely, the disorder effect is crucial to the PKE measurement[17].

Finally, we write down $\mathcal{L}'_{eff}[A_\mu]$ explicitly;

$$\begin{aligned} \mathcal{L}'_{eff}[A_\mu] = & \frac{1}{2\lambda_L(q)^2 v_F^2} A_0(q)^T A_0(-q)^T - \frac{1}{2\lambda_L(q)^2} A_i(q)^T A_i(-q)^T \\ & - \frac{a(q)}{2} i\epsilon_{ij} (A_0^T(q) q_i A_j^T(q) + A_i^T(q) q_j A_0^T(-q)) \\ & + \frac{b(q)}{2} i\epsilon_{ij} A_i^T(q) q_0 A_j^T(-q), \end{aligned} \quad (13)$$

where,

$$A_0^T(q) = A_0(q) - \frac{iq_0}{q_0^2 - v_F^2 \mathbf{q}_\perp^2} (q_0 A_0(q) - v_F^2 \mathbf{q}_\perp \cdot \mathbf{A}(q)), \quad (14)$$

$$A_i^T(q) = A_i(q) - \frac{iq_i}{q_0^2 - v_F^2 \mathbf{q}_\perp^2} (q_0 A_0(q) - v_F^2 \mathbf{q}_\perp \cdot \mathbf{A}(q)), \quad (15)$$

are the transversal components of gauge fields inside a media, namely, these are gauge invariant[8]. The current Eq. (1) in Ref. [17] is obtained with a gauge fixing condition $q_0 A_0(q) - v_F^2 \mathbf{q}_\perp \cdot \mathbf{A}(q) = 0$.

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